EDCA formal description

EDCA is a parallel bidirectional CA composition $\aleph = \aleph(\Omega, \Xi, \mathcal{I})$ where $\Omega = (d, S, \mathcal{N}, f)$ is the "ground" CA, $\Xi = \{\mathcal{E}_1, \mathcal{E}_2 \dots \mathcal{E}_K\}$ is a set of $K \in \mathbb{N}$ energy CA, each holding a spot, and $\mathcal{I}$ is the Energy Incidence Criterion. $\mathbb{Z}^d$ is a cellular space of dimension $d \in \mathbb{Z}_+$ with finite state set $S \subset \mathbb{Z}$. Configuration c: $\mathbb{Z}^d \to S$ is a function $c(\vec{n})$ that assigns state to each cell $\vec{n} \in \mathbb{Z}^d$.

$\mathcal{N} = (\vec{n}_1, \vec{n}_2, \dots, \vec{n}_p)$ −Neigborhood vector, where $\mathcal{N}(\vec{n}) = [\vec{n} + \vec{n}_1, \vec{n} + \vec{n}_2, \dots, \vec{n} + \vec{n}_p], p \in \mathbb{N}$.

State set of neighbors: $c[\mathcal{N}(\vec{n})] = \{c(\vec{n} + \vec{n}_1), c(\vec{n} + \vec{n}_2), \dots, c(\vec{n} + \vec{n}_p)\}$.

Function $f: S^p \to S$, $f(c[\mathcal{N}(\vec{n})])\ with\ f(\{0\}) = 0$ evaluates potential state change for cell $\vec{n}$ based on states of the neighbors.

Energy spots $\mathcal{E}_k = (d, Q, \zeta, h)$ share the same cellular space of dimension d with Ω, where spot index $k \in \mathbb{K} = \{1,2 \dots, K\}$, being $K \in \mathbb{N}$ a finite number of energy spots and $Q = \{-1,0,1\}$ − the state set for energy spot.

Configuration $\eta_k: \mathbb{Z}^d \to Q$ (energy coverage) is the function $\eta_k(\vec{n})$ that attaches a value for each energy CA $\mathcal{E}_k$ to cell $\vec{n}$. This function is also defined as the $k - energy$ state of the cell $\vec{n}$.

The set $\mathfrak{E}(k) \subset \mathbb{Z}^d$ is the energy coverage zone of the spot k, where $\mathfrak{E}(k) = \{\vec{n} | \eta_k(\vec{n}) \neq 0\}$. If the cell $\vec{n} \in \mathfrak{E}(k)$, it is said that the cell $\vec{n}$ *is covered by spot* k.

$\zeta = (\vec{n}_1, \vec{n}_2, \dots, \vec{n}_r)$, $r \in \mathbb{N}$, is the energy scope, where $\zeta(\vec{n}) = [\vec{n} + \vec{n}_1, \vec{n} + \vec{n}_2, \dots, \vec{n} + \vec{n}_r]$ defines the energy neighborhood of cell $\vec{n}$. State set of this neighborhood for each energy spot $\mathcal{E}_k$ is: $\eta_k[\zeta(\vec{n})] = \{\eta_k(\vec{n} + \vec{n}_1), \eta_k(\vec{n} + \vec{n}_1), \dots, \eta_k(\vec{n} + \vec{n}_r)\}$.

Energy spots evolve in time, each one as a separate CA. Spot evolution means that its energy coverage zone expands, thus reaching new cells at each iteration.

Energy expansion: $h(\eta_k[\zeta(\vec{n})])\ with\ h(\{0\}) = 0$, is the local update rule for energy spots. It is defined as a function $h: Q^r \to Q$, that evaluates expected energy coverage change $\eta_{k,t+1}(\vec{n})$, using the energy coverage of neighbors from the cell's $\vec{n}$ energy scope. Transformation $\mathfrak{E}_t(k) \longmapsto \mathfrak{E}_{t+1}(k)$ is a function $H: \mathbb{Z}^d \to \mathbb{Z}^d$, that evaluates the potential coverage zone change for the spot k. Function h may be generalized to become a 2nd order rule, eventually expressing its dependence on previous values $\eta_{k,t-1}(\vec{n})$. Vector $\vec{k} \in \mathbb{Z}^d$ is the origin of the spot k, that identifies

the starting point of its evolution. Starting coverage zone of spot k always matches its energy scope, that is: $\mathfrak{E}_0(k) = \zeta(\vec{k})$, having a unique coverage value for all cells:

$\eta_k(\vec{n}) = \eta_k(\vec{m}), \forall \vec{n}, \vec{m} \in \mathfrak{E}_0(k)$.

$\mathcal{E}_k$ orbits $\mathfrak{E}_0(k), \mathfrak{E}_1(k) \ldots \mathfrak{E}_T(k)$ are time cyclic, eventually returning to starting coverage zone $\mathfrak{E}_0(k) = \zeta(\vec{k})$ under certain conditions, having a new value of $\vec{k}$ calculated at the beginning of the cycle. Rule h is chosen to assure a unique value for all cells inside coverage zone, during a cycle T, that is: $\eta_k(\vec{n}) = \eta_k(\vec{m}), \forall \vec{n}, \vec{m} \in \mathfrak{E}_t(k), \forall t, 0 \leq t \leq T$.

Energy spot value v: $\mathbb{K} \to \{-1,1\}$ is the function $v(k) = \eta_k(\vec{n}) | \vec{n} \in \mathfrak{E}_0(k)$ that defines the spot type for a given cycle: negative or positive. Hence, the energy spot global configuration can be represented by the tuple: $e(k) = \langle \vec{k}, v, \mathfrak{E} \rangle$, that includes origin, value and coverage zone.

Definition 1: Spot k and cell $\vec{n}$ establish an *enabling relationship* if k covers $\vec{n}$ with a value that matches the sign of the expected cell transition. That is, spot k can contribute with its energy unit value to cell $\vec{n}$ for changing its state. Thus, the condition $\eta_k(\vec{n}) f(c[\mathcal{N}(\vec{n})]) > 0$ assures the enabling relationship between spot k and cell $\vec{n}$.

2.2.1. Energy Incidence Criterion

Let $\mathbb{E}_{\vec{n}} = \{k | \eta_k(\vec{n}) f(c[\mathcal{N}(\vec{n})]) > 0\}$ be the set of spots enabling cell $\vec{n}$. $\mathbb{A}_k = \{\vec{n} | k \in \mathbb{E}_{\vec{n}}\}$ is the set of all cells enabled by spot k. $\mathbb{E} = \bigcup_{\{\vec{n} | f(c[\mathcal{N}(\vec{n})]) \neq 0\}} \mathbb{E}_{\vec{n}}$ is the set of all spots enabling at least one cell, where $\mathbb{A} = \bigcup_{k \in \mathbb{K}} \mathbb{A}_k$ the set of all enabled cells. Sets $\mathbb{A}$ and $\mathbb{E}$ form the bipartite weighted graph $R = (\mathbb{A} \cup \mathbb{E}, L)$, where edge weights $L = \{\eta_k(\vec{n}) | \eta_k(\vec{n}) f(c[\mathcal{N}(\vec{n})]) > 0\}$ are the coverage values of all enabling relationships between sets $\mathbb{A}$ and $\mathbb{E}$.

For changing its state, the cell $\vec{n}$ must be *fully enabled,* that is, it needs to receive from spots $k \in \mathbb{E}_{\vec{n}}$, an amount of energy equal to its expected state change $f(c[\mathcal{N}(\vec{n})])$. Therefore, as a spot carries unitary amount of energy, each cell must be assigned a number $|f(c[\mathcal{N}(\vec{n})])|$ of spots in order to be updated, having each spot assigned to only one cell.

Some conflictive situations can arise here when a spot covers many cells, or an exceeding number of spots is willing to be assigned to a cell. To resolve these conflicts, graph R is transformed into a collision free Assignation Graph $\check{R} = (\check{\mathbb{A}} \cup \check{\mathbb{E}}, X)$, $\check{\mathbb{A}} \subseteq \mathbb{A}$, $\check{\mathbb{E}} \subseteq \mathbb{E}$ which fulfills the following conditions:

$$\forall \vec{n} \in \breve{\mathbb{A}}, \exists \breve{\mathbb{E}}_{\vec{n}} \subseteq \mathbb{E}_{\vec{n}} \,\big|\, f(c[\mathcal{N}(\vec{n})]) = \sum_{k \in \breve{\mathbb{E}}_{\vec{n}}} \eta_k(\vec{n}) \; - fully\ enabling\ cell\ \vec{n}$$

$$\left.\begin{array}{l} \breve{\mathbb{E}} = \cup_{\vec{n} \in \breve{\mathbb{A}}} \breve{\mathbb{E}}_{\vec{n}} \,, \breve{\mathbb{E}}_{\vec{n}} \cap \breve{\mathbb{E}}_{\vec{m}} = \emptyset \,, \forall \vec{m}, \vec{n} \in \breve{\mathbb{A}} \,, \vec{m} \neq \vec{n} \\ \breve{\mathbb{A}} = \bigcup_{k \in \breve{\mathbb{E}}} \breve{\mathbb{A}}_k \,, \breve{\mathbb{A}}_k = \{\vec{n} \in \mathbb{A}_k | k \in \breve{\mathbb{E}}_{\vec{n}}\}, where\ |\breve{\mathbb{A}}_k| = 1, \forall k \in \breve{\mathbb{E}} \end{array}\right\} \textit{Unitary spot contribution}$$

$$X = \{\eta_k(\vec{n}) | \vec{n} \in \breve{\mathbb{A}}_k, k \in \breve{\mathbb{E}}_{\vec{n}}\}$$

Definition 2: Energy Incidence Criterion $\mathcal{I}$ is the transformation $\mathcal{I}: R \longmapsto \breve{R}$ that defines assigned (selected for updating) cells and spots from enabling graph R. Inclusion of cell $\vec{n}$ and spot k in sets $\breve{\mathbb{A}}$ and $\breve{\mathbb{E}}$ respectively, determines how their next states and values are calculated. Consequently, EDCA evolution is determined:

I- By its startup ground CA Ω configuration $\mathrm{c}_0(\vec{n})$ and the set of K energy spot CA initial global configurations $\{e_0(k), k \in \mathbb{K}\}$, where $e_0(k) = \langle \vec{k}, v, \zeta(\vec{k}) \rangle$ are initial locations, values and coverage zone for each spot (energy CA $\mathcal{E}_k$).

II- By the global EDCA transition, which involves the following sequence of operations:

1-Transformation: $\mathcal{I}: R \longmapsto \breve{R}$, with $\breve{R} = (\breve{\mathbb{A}} \cup \breve{\mathbb{E}}, X)$

2-Transition $c_t \longmapsto c_{t+1}$, applied to the set $\breve{\mathbb{A}}$ of cells:

$$\mathrm{c}_{t+1}(\vec{n}) = c_t(\vec{n}) + f(c_t[\mathcal{N}(\vec{n})]), \;\; \forall\, \vec{n} \in \breve{\mathbb{A}}$$

3-Transition $e_t \longmapsto e_{t+1}$, applied to all spots:

$$e_{t+1}(k) = \begin{cases} \langle\, \vec{n}, -v, \zeta(\vec{n}) \rangle, & if\ \exists \vec{n} |\ k \in \breve{\mathbb{E}}_{\vec{n}} \\ \langle \vec{k}, v, H(\mathfrak{E}_t) \rangle\,, & if\ k \notin \breve{\mathbb{E}} \end{cases}$$

This update sequence is asynchronous, since only cells $\vec{n} \in \breve{\mathbb{A}}$ are updated at each time step. Transitions 2 and 3 are made on a strictly local basis, while $\mathcal{I}$ transformation is a global procedure employed for establishing the asynchronous update order of cells and spots.

Evolution of EDCA is restricted to finite configurations [1], with functions f and h having a quiescent state $f(\{0\}) = 0$, and $h(\{0\}) = 0$. At each iteration, cells having states $\mathrm{c}(\vec{n}) \neq 0$ and $\eta_k(\vec{n}) \neq 0$ form a finite set. Provided a finite set $\mathbb{K}$ of spots, we can conclude that sets $\mathbb{A} \subseteq \{\vec{n} | f(c[\mathcal{N}(\vec{n})]) \neq 0\}$ and $\mathbb{E} \subseteq \mathbb{K}$, and consequently, transformation $\mathcal{I}: R \longmapsto \breve{R}$ are also finite.